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Professor Teller raises the question: Can we be local physicalists? He reduces this question, in part at least, to the question: Are there ineliminably relational properties?, i.e. relational properties which do not supervene on the non-relational properties of individuals. In the first part of his paper he considers the case of classical physics. With the exception of position we can be local physicalists, but when we turn to quantum mechanics Teller argues 'No'. I think that this part of Teller's paper is too quick. This is because he does not distinguish with sufficient care between relational holism and nonlocality in the sense of action-at-a-distance between individuals. I want, in my own presentation, to explain two ways in which this distinction has been drawn in the recent literature on the philosophy of quantum mechanics.

(1) I begin with the work of Heywood and Redhead ([1]) on algebraic proofs of nonlocality.

Realist construals of quantum mechanics (QM) in which all observables possess sharp values at all times, have met with two major problems. The first is posed by the Kochen-Specker paradox ([2]) in which value assignments to appropriate observables subject to a constraint known as FUNC (Functional Composition Principle) lead to an algebraic contradiction. The second problem arises from the work of Bell ([3]) which shows that in the case of two spatially separated systems the correct QM correlations between the two systems can in general only be obtained by violating locality, i.e. assuming that the value of an observable pertaining to one system depends on what sort of measurement procedure is performed on the other system. Heywood and Redhead showed how these results are linked by giving a demonstration of nonlocality which does not involve consideration of correlation functions as in Bell's work, but

Environmental Contextuality recognizes that the values of these quantities may depend on the environment, in particular on the apparatus set to measure some maximal observable on the system.

We apply these notions to two spatially separated systems and arrive at two quite distinct locality principles.

OLOC: Locally maximal physical magnitudes on either of two spatially separated systems are not 'split' by ontological contextuality relative to the specification of different maximal physical magnitudes for the joint system.

ELOC: The values possessed by a local physical magnitude cannot be changed by altering the arrangement of a remote piece of apparatus which forms part of the measurement context for the joint system.

Note that in terms of measurement results OLOC and ELOC cannot be distinguished. The violation of either demonstrates a dependence of the outcome recorded by an apparatus connected to one system on the setting of the apparatus connected to the other (remote) system.

OLOC is not assumed in specifying ELOC, but ELOC is only properly a locality principle if OLOC obtains. This is because violation of OLOC means we cannot specify a locally maximal magnitude independently of properties relating to the whole combined system. This leads to an ontological holism in which it is impossible to make sense of a realist construal of QM that associates properties independently with each of two separated systems.

If OLOC is assumed then violation of ELOC is the sort of nonlocality envisaged in the Bell-type nonlocality arguments.

Assuming VR and the innocent FUNC\* we are forced then to

some form of nonlocality expressed in the violation of ELOC and/or OLOC. Violation of OLOC corresponds to Teller's relational holism, but we are not committed to that if we accept violation of ELOC.

(2) I turn now to the work of Jarrett ([5]) on stochastic hidden variable theories\*

We consider two quantities  $a$  and  $b$  which are local magnitudes for two spatially separated systems  $\alpha$  and  $\beta$ . Let  $\mathcal{E}_a$  and  $\mathcal{E}_b$  be possible values for these quantities. Let an apparatus A measure  $\mathcal{E}_a$  on  $\alpha$  and an apparatus B measure  $\mathcal{E}_b$  on  $\beta$ . Let the two systems emerge from a source S characterized in its complete state by a parameter (hidden variable)  $\lambda$ , with possible values  $\mathcal{E}_\lambda$ . Then we suppose the following 3-joint distribution is well-defined

$$\text{Prob}_{a,b,\lambda}^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_a, \mathcal{E}_b, \mathcal{E}_\lambda)$$

where  $\eta_A$ ,  $\eta_B$  represent the physical configuration of the apparatus A and B and  $\eta_S$  the configuration of the source.

We write

$$\begin{aligned} & \text{Prob}_{a,b,\lambda}^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_a, \mathcal{E}_b, \mathcal{E}_\lambda) \\ &= \text{Prob}_a^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_a / \mathcal{E}_b \& \mathcal{E}_\lambda) \\ & \times \text{Prob}_b^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_b / \mathcal{E}_\lambda) \\ & \times \text{Prob}_\lambda^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_\lambda) \end{aligned} \quad (1)$$

We now make a completeness assumption

$$\text{Prob}_a^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_a / \mathcal{E}_b \& \mathcal{E}_\lambda) = \text{Prob}_a^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_a / \mathcal{E}_\lambda) \quad (2)$$

The significance of (2) is that  $\mathcal{E}_\lambda$  is sufficient to determine completely  $\text{Prob}_a^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_a / \mathcal{E}_b \& \mathcal{E}_\lambda)$ .

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\*I have adapted some of Jarrett's definitions to suit my own arguments.

Specification of  $\mathcal{E}_b$  is not required. Suppose the completeness condition is violated. Then the probability distribution of properties possessed by the  $\alpha$  system, for a given state of the source, will depend in an essential way on what property is possessed by the  $\beta$  system. This could arise in two possible ways:

- (1) There is a stochastic causal link between the property  $a$  of  $\alpha$  and the property  $b$  of  $\beta$ .

To support such a claim we would have to suppose that any mechanism for changing  $\mathcal{E}_b$  would issue in a change in probability distribution for  $\mathcal{E}_a$ . This would be nonlocality in the sense of action-at-a-distance.

- (2) But we do not have to suppose a stochastic causal mechanism connecting local properties of the  $\alpha$  and  $\beta$  system. All that we can say is that  $\lambda$  does not screen off  $a$  from  $b$  in Reichenbach's terminology. This shows arguably that  $\lambda$  is not a common cause of  $a$  and  $b$ , but again arguably it is not sufficient to demonstrate a causal dependence between  $a$  and  $b$ . We could interpret the situation as evidence for a holism or nonseparability in the particular state in which  $\alpha$  and  $\beta$  emerge from the source. The  $\alpha$  system does not possess independent properties (propensities) of its own. The conditional probability  $\text{Prob}_a^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_a / \mathcal{E}_b \& \mathcal{E}_\lambda)$  is a candidate for an ineliminably relational property for the joint system  $\alpha + \beta$ . If we follow this proposal, violation of the completeness condition would parallel violation of OLOC in the Redhead-Heywood analysis.

Under the completeness assumption, Eq.(1) reduces to

$$\begin{aligned}
 & \text{Prob}_{a,b,\lambda}^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_a, \mathcal{E}_b, \mathcal{E}_\lambda) \\
 &= \text{Prob}_a^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_a / \mathcal{E}_\lambda) \\
 &\quad \times \text{Prob}_b^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_b / \mathcal{E}_\lambda) \\
 &\quad \times \text{Prob}^{\eta_A, \eta_B, \eta_S}(\mathcal{E}_\lambda)
 \end{aligned} \tag{3}$$

We can now make locality assumptions that parallel ELOC.  
Schematically

$$\begin{aligned}
 \text{Prob}_{\text{a}}^{\eta_A, \eta_B, \eta_S}(\epsilon_a / \epsilon_\lambda) \\
 &= \text{Prob}_{\text{a}}^{\eta_A, \eta_S}(\epsilon_a / \epsilon_\lambda) \\
 \text{Prob}_{\text{b}}^{\eta_A, \eta_B, \eta_S}(\epsilon_b / \epsilon_\lambda) \\
 &= \text{Prob}_{\text{b}}^{\eta_B, \eta_S}(\epsilon_b / \epsilon_\lambda) \\
 \text{Prob}^{\eta_A, \eta_B, \eta_S}(\epsilon_\lambda) &= \text{Prob}^{\eta_S}(\epsilon_\lambda)
 \end{aligned} \tag{4}$$

where we suppress the parameter on which the indicated probability distribution does not depend.

Then (3) reduces to

$$\begin{aligned}
 \text{Prob}_{\text{a}, \text{b}, \lambda}^{\eta_A, \eta_B, \eta_S}(\epsilon_a, \epsilon_b, \epsilon_\lambda) \\
 &= \text{Prob}_{\text{a}}^{\eta_A, \eta_S}(\epsilon_a / \epsilon_\lambda) \\
 &\times \text{Prob}_{\text{b}}^{\eta_B, \eta_S}(\epsilon_b / \epsilon_\lambda) \\
 &\times \text{Prob}_{\lambda}^{\eta_S}(\epsilon_\lambda)
 \end{aligned} \tag{5}$$

But with the representation (5) one can show that the Bell inequality is satisfied for appropriate choice of the properties a and b. Since this is contradicted by experiment we can impute either (2) or (4).

Violation of (4) shows a clear case of action-at-a-distance. Changing the setting of the apparatus B changes conditional probabilities of properties manifested at locations A and S, and so on.

Violation of (2), as we have seen can be interpreted in terms of action-at-a-distance, but an alternative interpretation is possible in terms of ineliminably relational properties. So once again the metaphysical possibilities in the quantum-mechanical case appear to be richer than Professor Teller allows.

# References

- [1] Heywood, P and Redhead, M.L.G. "Nonlocality and the Kochen-Specker Paradox", Foundations of Physics 13 (1983) 481-499.
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